You *fuzzyin'* with me?

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"So far as the laws of mathematics refer to reality, they are not certain. And so far as they are certain, they do not refer to reality." -Albert Einstein, Geometry and Experience

Abstract

In the movie *2001:A Space Odyssey*, Stanley Kubrick predicted how fast the computer technology would grow, by introducing the mainframe HAL, a "smart" computer with the ability to think and learn. Though, it is highly unlikely, that 5 years from now, man would have created HAL, one thing is for sure, it will eventually come from Japan. Why? Because of a very simple little thing in which the Japanese have adapted for decades; the Americans sweating to catch up;- fuzzy logic. This article introduces fuzzy logic, from a mathematics perspective, tracing its origins, stating its characteristics and finally illustrating with examples, its set theories and logic operations.

Fuzzy *what* ??!!

Fuzzy logic is a superset of conventional(Boolean) logic that has been extended to handle the concept of partial truth- truth values between "completely true" and "completely false". As its name suggests, it is the logic underlying modes of reasoning which are approximate rather than exact. The importance of fuzzy logic derives from the fact that most modes of human reasoning and especially common sense reasoning are approximate in nature.

Boolean vs. Fuzzy

300 years B.C., the Greek philosopher, Aristotle came up with binary logic(0,1), which is now the principle foundation of Mathematics. It came down to one law: A or not-A, either this or not this. For example, a typical rose is either red *or* not red. It cannot be red *and* not red. Every statement or sentence is true or false or has the truth value 1 or 0. This is Aristotle's law of bivalence and was philosophically correct for over two thousand years.

Two centuries before Aristotle, Buddha, had the belief which contradicted the black-and-white world of worlds, which went beyond the bivalent cocoon and see the world as it is, filled with contradictions, with things *and* not things. He stated that a rose, could be to a certain degree completely red, but at the same time could also be at a certain degree not red. Meaning that it can be red *and* not red at the same time. Conventional(Boolean) logic states that a glass can be full *or* not full of water. However, suppose one were to fill the glass only halfway. Then the glass can be half-full *and* half-not-full. Clearly, this disprove's Aristotle's law of bivalence. This concept of *certain degree* or multivalence is the fundamental concept which propelled Zader Lofti of University Berkely in the 1960's to introduce fuzzy logic. The essential characteristics of fuzzy logic founded by him are as follows.

- In fuzzy logic, exact reasoning is viewed as a limiting case of approximate reasoning.
- In fuzzy logic everything is a matter of degree.
- Any logical system can be fuzzified
- In fuzzy logic, knowledge is interpreted as a collection of elastic or, equivalently , fuzzy constraint on a collection of variables
- Inference is viewed as a process of propagation of elastic constraints.

The third statement hence, define Boolean logic as a subset of Fuzzy logic.

Fuzzy Subset Theory

There is a strong relationship between Boolean logic and the concept of a subset. Similarly there is a strong relationship between fuzzy logic and fuzzy subset theory.

A subset U of a set S can be defined as a set of ordered pairs, each with a first element that is an element of the set S, and a second element that is an element of the set { 0, 1 }, with exactly one ordered pair present for each element of S. This defines a mapping between elements of S and elements of the set { 0, 1 }. The value zero is used to represent non-membership, and the value one is used to represent membership. The truth or falsity of the statement

x is in U

is determined by finding the ordered pair whose first element is x. The statement is true if the second element of the ordered pair is 1, and the statement is false if it is 0.

Similarly, a fuzzy subset F of a set S can be defined as a set of ordered pairs, each with a first element that is an element of the set S, and a second element that is a value in the interval [0, 1], with exactly one ordered pair present for each element of S. This defines a mapping between elements of the set S and values in the interval [0, 1]. The value zero is used to represent complete non-membership, the value one is used to represent complete membership, and values in between are used to represent intermediate *degrees of membership*. The set S is referred to as the *universe of discourse* for the fuzzy subset F. Frequently, the mapping is described as a function, the *membership function* of F. The degree to which the statement

x is in F

is true is determined by finding the ordered pair whose first element is x. The *degree of truth* of the statement is the second element of the ordered pair.

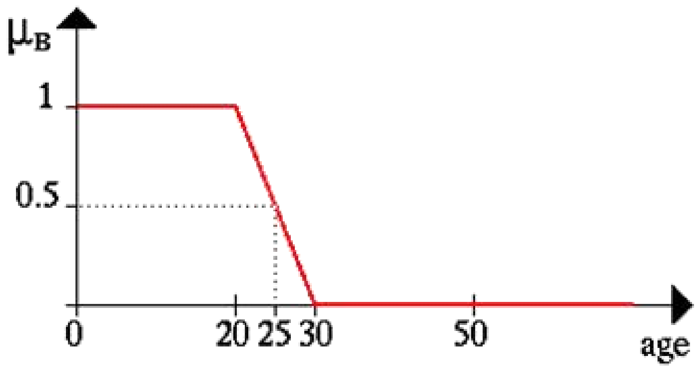
This can be illustrated with an example. Let's talk about people and "youthness". In this case the set S (the universe of discourse) is the set of people. A fuzzy subset YOUNG is also defined, which answers the question "to what degree is person x young?" To each person in the universe of discourse, we have to assign a degree of membership in the fuzzy subset YOUNG. The easiest way to do this is with a membership function based on the person's age.

young(x) = { 1, if age(x) <= 20.,

(30-age(x))/10, if 20 < age(x) <= 30,

0, if age(x) > 30 }

A graph of this looks like:



Given this definition, here are some example values:

Person	Age	degree of youth
Johan	10	1.00
Edwin	21	0.90
Parthiban	25	0.50
Arosha	26	0.40

Chin Wei	28	0.20
Rajkumar	83	0.00

So given this definition, we'd say that the degree of truth of the statement "Parthiban is YOUNG" is 0.50.

Note: Membership functions almost never have as simple a shape as age(x). They will at least tend to be triangles pointing up, and they can be much more complex than that. Furthermore, membership functions so far is discussed as if they always are based on a single criterion, but this isn't always the case, although it is the most common case. One could, for example, want to have the membership function for YOUNG depend on both a person's age and their height (Arosha's short for his age). This is perfectly legitimate, and occasionally used in practice. It's referred to as a two-dimensional membership function. It's also possible to have even more criteria, or to have the membership function depend on elements from two completely different universes of discourse.

Fuzzy Logic Operations

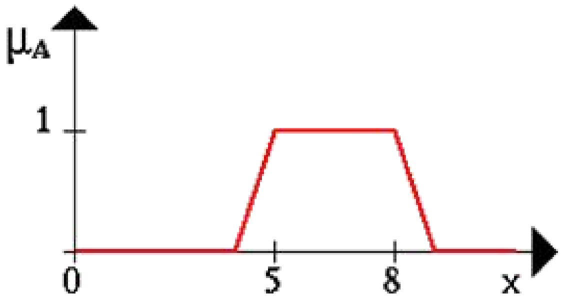
It is clear what the statement *X is LOW* means in fuzzy logic. But, how do we interpret a statement like

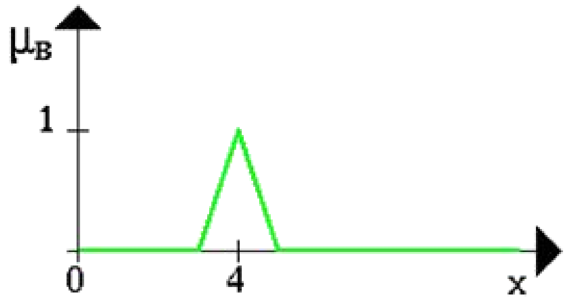
X is LOW and Y is HIGH or (not Z is MEDIUM)

The standard definitions in fuzzy logic as suggested by Lotfi are:

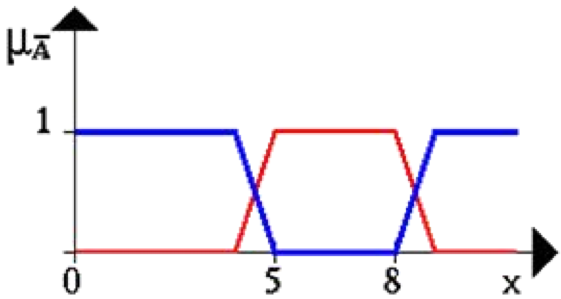
1) *Negate(negation criterion) : truth (not x) = 1.0 - truth (x)*
2) *Intersection(minimum criterion): truth (x and y) = minimum (truth(x), truth(y))*
3) *Union(maximum criterion): truth (x or y) = maximum (truth(x), truth(y))*

In order to clarify this, a few examples are given. Let *A* be a fuzzy interval *between* 5 *and* 8 and *B* be a fuzzy number *about* 4. The corresponding figures are shown below.

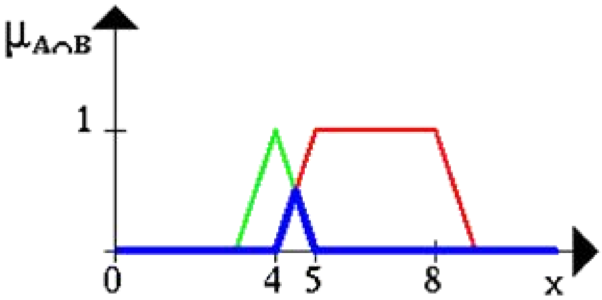




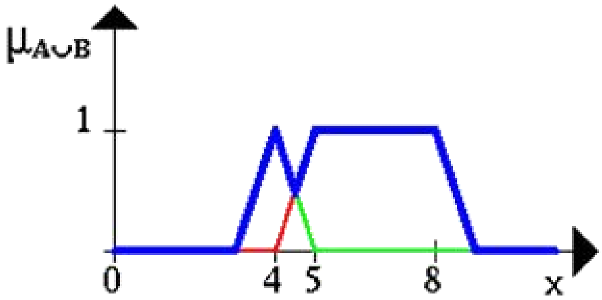
The figure below gives an example for a negation. The blue line is the **NEGATION** of the fuzzy set *A*. Note that the *negation* criterion is used.



The following figure shows the fuzzy set *between* 5 *and* 8 **AND** *about* 4 (blue line). This time the *minimum* criterion is used.



Finally, the Fuzzy set *between* 5 *and* 8 **OR** *about* 4 is shown in the next figure (blue line). This time the *maximum* criterion is used.



These basic operations, provide guidelines to construct more complex ones which in turn can be used to create fuzzy machines.

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